Gov 2002 Section 4 Notes
Fixed FX and Diff-in-Diff

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1 Fixed Effects estimation

Setup: multiple groups of observations (individuals, states, countries, etc) each with across-time variability OR multiple groups of observations (states, countries, etc.) each with sub-group variability

How to describe it: we're estimating the treatment effect by looking at variability within each group

Non-technical assumptions for identification: you must control for any confounders that vary within a group

Automatically controls for: any covariates that are fixed at the group level and impact all the units within a group equally

Estimation:

- within-estimator - used with time or sub-group variability
- first-difference estimator - used with time variability
- dummy variables for the fixed effects - can be used with either setup - can be challenging if you have tons of groups since it'll estimate a coefficient for each group, as well as the variance-covariance matrix which will grow quadraticly as the number of groups increase

Estimating via the within-estimator

Define the "within transformation function" as $\ddot{Z}_{it} = Z_{it} - \overline{Z}_i$

Use this function to average the outcome, treatment, and covariates across time (or county/zipcode if your groups are states)

$$\ddot{Y}_{it} = \ddot{X}'_{it}\beta + \tau\ddot{D}_{it} + \ddot{\epsilon}_{it}$$

where:

$$\begin{aligned}
\ddot{Y}_{it} &= Y_{it} - \overline{Y}_i \\
\ddot{X}_{it} &= X_{it} - \overline{X}_i \\
\ddot{D}_{it} &= D_{it} - \overline{D}_i \\
\ddot{\epsilon}_{it} &= \epsilon_{it} - \overline{\epsilon}_i
\end{aligned}$$

Once you've done the transformations, you can use OLS $[(X'X)^{-1}X'y]$ to estimate β and τ

Remember that unless you use a package in R or Stata that does a correction, your standard errors will be wrong when using the within estimator

2 Two-way fixed effects models

Estimating via the within-estimator

Start with $\ddot{Y}_{it}$, which is the value of the outcome for each observation (Y_{it}) with the group mean for that observation ($\overline{Y}_i$) subtracted out

Now lets apply the within transformation a second time to subtract off the time variability:

$$\begin{aligned}
\ddot{Y}_{it} &= Y_{it} - \overline{Y}_i \\
\ddddot{Y}_{it} &= (Y_{it} - \overline{Y}_i) - \overline{(Y_{it} - \overline{Y}_i)} \\
&= (Y_{it} - \overline{Y}_i) - \frac{1}{N}\sum_{i=1}^{N}(Y_{it} - \overline{Y}_i) \\
&= (Y_{it} - \overline{Y}_i) - \frac{1}{N}\sum_{i=1}^{N}Y_{it} + \frac{1}{N}\sum_{i=1}^{N}\overline{Y}_i \\
\ddddot{Y}_{it} &= Y_{it} - \overline{Y}_i - \overline{Y}_t + \overline{Y}
\end{aligned}$$

$$\ddddot{Y}_{it} = \ddddot{X}'_{it}\beta + \tau\ddddot{D}_{it} + \ddot{\epsilon}_{it}$$

Then you can apply the same formula to D and X and use regression to estimate the effects. Blocked bootstrapping or clustering will get the correct SEs

3 Diff-in-diff

Setup: multiple groups of observations with across-time variability

How to describe it: compare the average change over time in the outcome for the treated group to the average change over time for the controls

Non-technical assumptions for identification: parallel trends - the trend in the change over time for the treated group would look similar to the trend for the control group, had the treated units been controls

Imagine this is your data setup:

Y_{it}	$i = 0$	$i = 1$
$t = 0$	$y_{0,0}$	$y_{1,0}$
$t = 1$	$y_{0,1}$	$y_{1,1}$

The diff-in-diff estimator then is $(y_{1,1} - y_{0,1}) - (y_{1,0} - y_{0,0})$

Notice that if we distribute the negative sign, the estimator looks similar to the two-way fixed effects transformation:

$$y_{1,1} - y_{0,1} - y_{1,0} + y_{0,0} \approx Y_{it} - \overline{Y}_i - \overline{Y}_t + \overline{Y}$$

Estimating diff-in-diffs with regression

$$y = \beta_0 + \beta_1 T + \beta_2 I \beta_3 (T * I) + \epsilon$$

If this is out setup then:

$$\begin{aligned}
\hat{\beta}_0 &= E(y|T = 0, I = 0) \\
\hat{\beta}_1 &= E(y|T = 1, I = 0) \\
\hat{\beta}_2 &= E(y|T = 0, I = 1) \\
\hat{\beta}_3 &= E(y|T = 1, I = 1) - E(y|T = 0, I = 1) - E(y|T = 1, I = 0) + E(y|T = 0, I = 0)
\end{aligned}$$

Thus, $\hat{\beta}_3$ is corresponds to the formula for the diff-in-diff above $(y_{1,1} - y_{0,1} - y_{1,0} + y_{0,0})$