

DAGs and Potential Outcomes

Gov 2002 Section

September 15, 2015

Problem Set

First problem set is posted on the Assignments page on Canvas.

It's due by 5:00pm next Friday (September 25). Upload your answers as a PDF to Canvas.

If you're auditing you should still be able to download the assignment, but don't submit answers because I won't grade them.

Don't forget: $V(X) = E(X^2) - E(X)^2$

Outline

1 Potential Outcomes

2 DAGs

Basics of potential outcomes

- A potential outcome is the value of an outcome variable given a particular treatment condition
- Think of them as fixed, pre-treatment covariates. The only catch is that we only ever get to observe the one that corresponds with the treatment that's given (fundamental problem of causal inference)
- $Y_i(0)$ and $Y_i(1)$ denote the potential outcomes for unit i given control or treatment
- The observed outcome Y_i can be expressed as a function of the treatment and potential outcomes:

$$Y_i = D_i \cdot Y_i(1) + (1 - D_i) \cdot Y_i(0)$$

$Y_i(0)$ and $Y_i(1)$ are both fixed constants. D_i is a random variable. Therefore Y_i is a random variable.

Stable Unit Treatment Value Assumption

- Also known as the consistency assumption
- The observed outcome under a particular treatment condition **is** the potential outcome under that treatment. You typically need to make this assumption in order to make causal claims.

Stable Unit Treatment Value Assumption

SUTVA implies that:

- there is *no interference* between units:

$$Y_i(d_1, d_2, d_3, \dots) = Y_i(d_i)$$

In other words, whether or not one person received treatment or control does not affect the value of a different person's Y

- there is no variation in the treatment conditions (or that any variation is irrelevant)

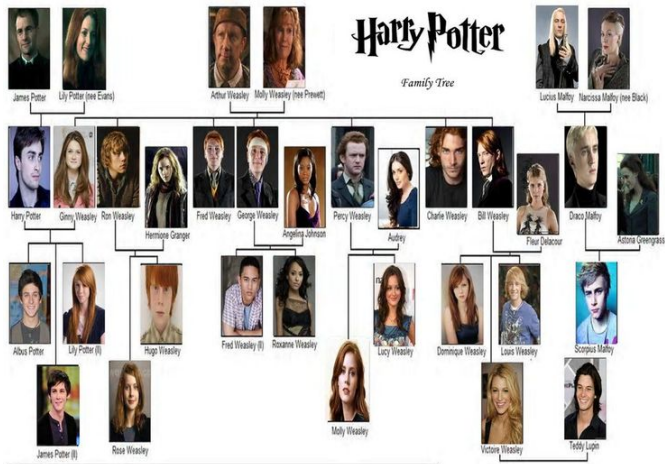
If the treatment condition is that you take one pill of Aspirin, you shouldn't have some units taking a 50mg pill and others taking a 200mg pill. If you do then you either need to consider those two separate treatment categories or you need to assume that the difference between a 50mg and a 200mg pill is irrelevant

Outline

1 Potential Outcomes

2 DAGs

You've seen DAGs before



What's a DAG?

DAG terminology:

- Node or vertex: the 'variables' or endpoints within the DAG
- Edge: the relationship between two nodes in the DAG. If there is a relationship between nodes A and B then there will be an arrow connecting the two of them. If there's no relationship between A and B then that edge will be blank
- Path: Any set of arrows that starts at node A and ends at node B, which does not visit any single node more than once. It doesn't matter the direction the arrows are pointing.
 - Causal path: A path between A and B is causal if all the arrows along the path point in the same direction.
- Parent: A is a parent of B if there's a path that ends at B and includes A.
- Descendant/child: The opposite of a parent. B is a descendant of A if there's a path which ends at B and includes A.

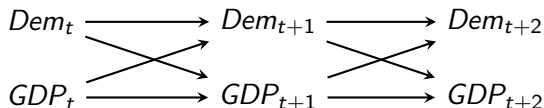
Requirements for a DAG to be a DAG

- 1 Must be directed - relationships between two nodes must be unidirectional. A cannot cause B while B also causes A .

This is only logical—if A causes B then A must occur before B . Therefore B cannot cause A because it happened after A .

But what about 'reciprocal relationships'? Doesn't growth in GDP cause democratization and democratization cause growth in GDP?

Yes, but over multiple time periods:



Requirements for a DAG to be a DAG

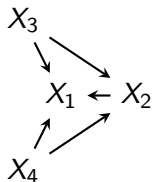
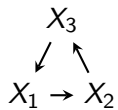
- 2 No cycles—a variable cannot cause itself
- 3 Causal Markov assumption - conditional on its direct causes (i.e. parents) a variable is independent of its non-descendants

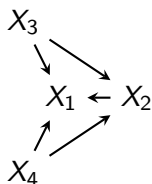
In other words, if we control for everything that directly causes Y then Y will be independent/uncorrelated of everything else in the world that happened before it

Are these DAGs?

$$X_1 \rightarrow X_2$$

$$X_1 \leftrightarrow X_2$$





What are all the paths between X_1 and X_2 ?

$$X_1 \leftarrow X_2$$

$$X_1 \leftarrow X_3 \rightarrow X_2$$

$$X_1 \leftarrow X_4 \rightarrow X_2$$

Which of these paths are causal?